A Comment on Transfers to Sustain Cooperation in a Dynamic Game of Climate Change

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Germain et al. (2003) study a dynamic model of climate change and propose a solution for which they claim to prove analytically a core property. The purpose of this note is to point out that they do not actually prove analytically any such result and their solution may not satisfy subgame perfection despite the fact that it is the outcome of what they call backward induction.

To begin with, Germain et al. (2003) claim that their model admits a unique feedback Nash equilibrium without proving the same (last line, p.89). This claim is false and based on the wrong belief that the conditions for the existence of a feedback Nash equilibrium of a dynamic game and for the existence of a Nash equilibrium of a strategic game are the same. In fact, a feedback Nash equilibrium in similar dynamic games has not been so far shown to exist except for quadratic cost and damage functions (see e.g. Dockner and Long, 1993 and Mäler and de Zeeuw, 1998). Since Germain et al. (2003) take for granted the existence of a feedback Nash equilibrium, the rest of their analysis is also questionable. Indeed they note that "…the problem is not analytically tractable…" in contradiction to the claim in the abstract to their paper that "… a core property is proved analytically … ." In fact, they neither provide a formal definition of their solution concept nor prove analytically any result.

To be precise, Germain et al. (2003) in total has *only* one formally stated result (stated as "Theorem" on page 89), but does not prove even that one and refers instead to an unpublished paper for the proof. Moreover, that proof is also not correct. First, their condition (29) cannot be shown to hold for *all* values of the GHG stock. This is necessary because in a dynamic game, the GHG stock can take any value whereas the referred proof holds, if at all, only for a fixed GHG stock. Second, their definition of the "value functions" $\widetilde{W}_i$ (see eq. (25)) is circular in that one of the equations which define the function $\widetilde{W}_i$ itself is defined by the function $\widetilde{W}_i$ (see eq. (28)). They also leave unspecified the conditions under which their endogenously defined value function $\widetilde{W}_i$ is indeed convex in each period, even though the authors are aware that it is not convex under their assumptions.

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Germain et al. (2003) argue that there exists a sequence of strategic games each of which admits a nonempty γ-core (Chander and Tulkens, 1997) if the so-called value function $\widetilde{W}_i$ is assumed to be convex in each period, but they treat each strategic game in the sequence in isolation and neither show how the strategic games in the sequence are related nor show how the γ-core allocations of these strategic games constitute a solution of the dynamic game. Thus, the so-defined solution may not satisfy subgame perfection. In particular, it may not be immune to the following strategy of a coalition: the coalition decides to not cooperate with the remaining players in *any* future period and thereby guarantee itself a payoff which is equal to its payoff in the feedback Nash equilibrium in which the coalition acts as one single player and the remaining players act as singletons. Chander and Tulkens (1997) refer to this type of equilibrium as a partial agreement equilibrium, except that the game is now dynamic, and define the γ-core as the set of all feasible allocations which are immune to such strategies of coalitions. Though Germain et al. (2003) rely on the γ-core concept, they do not show that their solution is immune to the dynamic analog of partial agreement equilibrium strategies. Thus, their solution concept does not rule out that a country or coalition of countries may have incentives to break away from the grand coalition in some period and follow instead its feedback Nash equilibrium strategies in all future periods.

Though backward induction, under some conditions, is known to lead to a subgame perfect equilibrium outcome in non-cooperative games, the same has not been shown to be the case for dynamic games in which players may cooperate and form the grand coalition and share the resulting surplus at each stage of the game. Thus, applying a cooperative version of backward induction does not imply that the resulting outcome is subgame perfect. It has to be shown that it is, especially since the method of backward induction, as applied by Germain et al. (2003), amounts to modifying the payoff *functions* of the players at each stage of the dynamic game. In a forthcoming paper, I discuss this issue and introduce a new solution concept for a similar dynamic game which, by definition, satisfies subgame-perfection.

References

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