WILL AN OLIGOPOLY BECOME A MONOPOLY?*

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Abstract

This paper is concerned with an old question: Will the firms in an oligopoly have incentives to merge to monopoly and will the monopoly be stable? To answer this question, we motivate and introduce a new core concept for a general partition function game and then prove stability of the merger-to-monopoly by applying the new concept to an oligopoly in partition function game form. Unlike previous core concepts, the new core concept, labelled the strong-core, neither assumes formation of any particular partition subsequent to a deviation nor imposes an exogenous criterion for selecting among the partitions that can possibly form. The paper shows that an oligopoly with any number of homogeneous firms without capacity constraints admits a nonempty strong-core and so does an oligopoly of not necessarily homogeneous firms with capacity constraints equal to their Nash equilibrium outputs. These results imply that an oligopoly will become a stable monopoly both in the long and short runs, unless prevented by law.

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1. Introduction

Numerous studies have focused on conditions under which horizontal mergers (i.e. forming coalitions of two or more firms) in a Cournot oligopoly can be profitable for participating firms. Notable contributions include Salant et al. (1983), Perry and Porter (1985), Deneckere and Davidson (1985), and Farrell and Shapiro (1990), among others. Most of these studies focus on incentives of oligopolistic firms to merge to monopoly. In particular, Deneckere and Davidson conclude: "… short of antitrust policy, the industry would concentrate almost completely towards monopoly." However, the question of stability of mergers, especially of the merger-to-monopoly, has been somewhat ignored, except by Rajan (1989).

In this paper I argue that mere profitability of a merger does not guarantee that the merger will be stable. Although oligopolistic firms do have an incentive to merge into a single multi plant firm because of higher monopoly profits, but still some firm or coalition of firms may decide not to merge to monopoly as it may conclude that its greatest profit potential lies in remaining separate. Thus, the grand coalition, though profitable, can be stable only if the monopoly profits can be split in a way that no firm or coalition of firms will have incentive to leave the grand coalition and become an independent market player. As will be shown, this depends on whether or not the core of the oligopoly game is nonempty. Indeed, Maskin (2003, p.3) asserts that if there are games in which the grand coalition may not be stable it is *only* among coreless games that we will find them. In other words, a nonempty core is a sufficient condition for stability of the grand coalition and an empty core is a necessary condition for its instability.

This means that if an oligopoly game admits a nonempty core, then we need to look no further as that implies that the merger-to-monopoly will be stable. But the answer is a lot more complicated than this because the payoff of a coalition leaving the grand coalition in an oligopoly game, unlike a characteristic function game, depends not only on the actions taken by it, but also on the reaction of the firms left behind. Accordingly, all existing core concepts for oligopoly games make one or another *ad hoc* assumption regarding the partition (of firms into coalitions) that may form subsequent to a deviation from the grand coalition. Hence, there are many definitions of the core depending on the assumption made regarding the reaction of the firms left behind – the so-called "market's ethos" in Rajan (1989). One simple definition of the core is given by assuming that subsequent to a deviation by a coalition from the grand coalition, the remaining firms break apart into singletons (Chander and Tulkens, 1997). Another equally simple definition is given by assuming that the remaining firms form one single coalition of their own (Maskin, 2003). However, none of these core concepts, though used in many applications, has yet become widely accepted as *the* core concept for partition function games mainly because they make one or another *ad hoc* assumption regarding the partition that may emerge subsequent to a deviation. Similar criticisms can be made against the classical α- and β- cores that not only assume formation of a single coalition by the players left behind, but also assume predatory behaviour on their part with no concern how this may affect their own payoffs.

The paper formulates an oligopoly as a strategic game and shows that it can be converted into a unique partition function game (Thrall and Lucas, 1963) by proving existence of a unique Nash equilibrium for each induced game in which each coalition in each partition of firms in the industry acts as one single player and the payoff to each coalition in a partition is the sum of profits that accrue to each of its members. As will be made clear in Section 3.1, the basic approach in the paper is same as in Salant et al. (1983) in that outputs, prices, and profits are determined endogenously in exactly the same way. Only the question addressed is different. Thus the analysis in this paper complements that in Salant et al. (1983) among others.

Since an oligopoly game can be converted into a partition function game, the paper motivates and introduces a new core concept in terms of a general partition function game and then applies it to the partition function game form of the oligopoly game. Unlike previous core concepts, this new concept, labelled the strong-core, neither assumes formation of any particular partition nor imposes an exogenous criterion for selecting among the partitions that can possibly form subsequent to a deviation. Thus, the strong-core seems to nicely settle the long standing debate on which core concept to use for partition function games in general and oligopoly games in particular. To be specific, the strong-core is the set of all payoff vectors that are feasible for the grand coalition and such that for every deviating coalition and every partition containing the deviating coalition that may possibly form either the deviating coalition is worse-off or some non-singleton coalition in the partition is worse-off.

The strong-core, by definition, does not rule out formation of any partition subsequent to a deviation except that the partition must include the deviating coalition. It is consistent with the traditional core in the sense that the strong-core reduces to the traditional core if the worth of every coalition is independent of the partition to which it belongs and the game is adequately represented by a characteristic function. As will be seen, it is also nicely related to the familiar γ- and δ- cores in that the strong core is generally smaller than the γ-core and for games with "positive externalities" including the oligopoly games, the strong-core is larger than the δ-core.

After introducing and justifying the strong-core for a general partition function game, I show that an oligopoly game admits a nonempty strong-core. More specifically, I show that an oligopoly with any number of homogenous firms without capacity constraints and increasing marginal costs of production admits a nonempty strong-core. I interpret the unique Nash equilibrium of an oligopoly as the equilibrium that may prevail if a strong antitrust policy is in place. I then show that if each firm in an oligopoly faces a capacity constraint equal to its Nash equilibrium output, then the corresponding partition function form of the oligopoly is partially superadditive and, therefore, the strong-core of an oligopoly with capacity constraints is equal to the γ-core and is nonempty. Thus, both in the long run when production capacities can be expanded and in the short run when they cannot be, an oligopoly will turn into a stable monopoly unless an antitrust policy is in place.

Since the strong-core seems to be of general interest, the paper offers a number of interpretations and justifications for it. For one, it introduces a notion of farsighted dominance and interprets and justifies the strong-core by showing that the strong-core payoff vectors are such that deviations from a strong-core payoff vector are farsightedly deterred by the strong-core payoff vector itself. That is in any partition that may possibly form subsequent to a deviation the deviating coalition is either immediately worse-off or not farsightedly better-off.

The contents of this paper are as follows: Section 2 motivates and introduces the strong -core and compares it with other core concepts. It shows that a well-known class of symmetric games admit nonempty strong-cores. Section 3 introduces a model of an oligopoly and proves that an oligopoly with any finite number of homogeneous firms and increasing marginal cost of production admits a nonempty strong-core and so does an oligopoly with any finite number of not necessarily homogeneous firms if each firm faces a capacity constraint equal to its Nash equilibrium output. Section 4 introduces a notion of farsightedness and offers an additional justification/interpretation for the strong core. Section 5 draws the conclusion.

2. Core concepts for partition function games

The purpose of this section is to first review the existing core concepts and then motivate and introduce a new core concept, to be called the strong-core, in the framework of a general partition function game. The next section applies this concept to an oligopoly by converting the oligopoly into a partition function game.

Let $N = \{1, \dots, n\}, n \geq 3$, denote the set of players. A set $P = \{S_1, \dots, S_m\}$ is a partition of N if $S_i \cap S_j = \emptyset$ for all $i, j = 1, \dots, m, i \neq j$, and $\cup_{i=1}^{m} S_i = N$. I shall denote the finest partitions of $N, S,$ and $N\backslash S$ by $[N], [S],$ and $[N\backslash S]$, respectively, the cardinality of set S by $|S|$, and (to save on notation) the singleton sets $\{i\}, \{S\}, \{N\backslash S\}$, and $\{N\}$ simply by $i, S, N\backslash S$, and N, respectively, whenever no confusion is possible.

A partition function is a real valued function of a coalition and a partition and denoted by $v(S; P)$ where P is a partition of N and S is a member of P. We shall denote a partition function game by a pair (N, v). Since the worth of a coalition in a partition function game depends on the partition to which the coalition belongs, the partition function games are sometimes referred to as games with externalities. A partition function game in which the worth of every coalition is independent of the partition and depends *only* on the coalition can be considered as a special case and adequately represented by a characteristic function.

Given a partition function game (N, v), a payoff vector $x = (x_1, \dots, x_n)$ is feasible for the grand coalition if $\sum_{i \in N} x_i = v(N; \{N\})$. I shall denote a payoff vector that is feasible for the grand coalition simply by (x, N). Similarly, a payoff vector $y = (y_1, \dots, y_n)$ is feasible for a *partition* $P = \{S_1, \dots, S_m\}$ if $\sum_{k \in S_i} y_k = v(S_i; P), i = 1, \dots, m$. Thus, in a payoff vector that is feasible for a partition, the payoff of each coalition in the partition is equal to its worth in the partition. I assume throughout the paper that each coalition in a partition is free to decide its part of the feasible payoff vector.

I shall denote a payoff vector that is feasible for a partition $P \neq N$, simply by $(y, P), P \neq N$, and, as a tie-breaker rule, henceforth adopt the convention that the players *strictly* prefer to be members of the grand coalition than of a coalition in a partition other than the grand coalition even if their payoffs are the same, i.e., player i is "better-off" as a member of the grand coalition with feasible payoff vector x than as a member of a coalition in a partition

$P \neq N$ with feasible payoff vector y if $x_i \geq y_i$. But if $P = N$, i.e., x and y are both feasible for *the grand coalition* and $x_i = y_i$, then player i is indifferent and not better-off.

2.1 *A new core concept*

The core, proposed by Gillies (1953) is a leading and influential solution concept for characteristic function games. But in a partition function game, unlike a characteristic function game, a deviating coalition has to take into account what other coalitions may form in the complement subsequent to its deviation, since its payoff/worth depends on the entire partition. Accordingly, all existing core concepts for a partition function game without fail make one or other *ad hoc* assumption concerning the coalitions that may form in the complement subsequent to a deviation – leading to alternative core concepts depending on the assumption made in this regard. In this subsection, I first review the two most widely used core concepts and then motivate and introduce a new core concept, to be called the strong-core of a partition function game, which does not assume formation of any particular partition subsequent to a deviation.

Definition 1 The γ-core of a partition function game (N, v) is the set of all payoff vectors (x, N) such that for every deviating coalition S and partition $\{S, [N\backslash S]\}$, $\sum_{i \in S} x_i \geq v(S; \{S, [N\backslash S]\})$.

The γ-core (Chander and Tulkens, 1997), motivated and introduced as an improvement over the classical α- and β- cores, assumes formation of a specific partition subsequent to a deviation from the grand coalition. In particular, it assumes that if coalition S deviates from the grand coalition then the partition $\{S, [N\backslash S]\}$ forms, and a γ-core payoff vector is such that the deviating coalition S is worse-off in this partition. But why should the complement of a deviating coalition break apart into singletons and not into some other partition?

Definition 2 The δ-core of a partition function game (N, v) is the set of all payoff vectors (x, N) such that in every deviating coalition S and binary partition $\{S, N\backslash S\}$, $\sum_{i \in S} x_i \geq v(S; \{S, N\backslash S\})$.

The δ-core (Maskin, 2003), like the γ-core, also assumes formation of a specific partition subsequent to a deviation from the grand coalition. In particular, if coalition S deviates from the grand coalition then the binary partition $\{S, N\backslash S\}$ forms, and a δ-core payoff vector is such that the deviating coalition S is worse-off in this partition. But notice that the δ-core actually requires not only the deviating coalition S but also the complementary coalition $N\backslash S$ to be worse-off, since a deviation by $N\backslash S$ from the grand coalition would result in the binary partition $\{N\backslash S, S\}$ in which the coalition $N\backslash S$, by definition of the δ-core, must be worse-off.

It is worth noting that the γ- and δ- cores, as defined above, are not the same as similarly named concepts in Hart and Kurz (1983). In contrast to the approach in the present paper, Hart and Kurz (1983) do not require the sum of payoffs of the members of a coalition in a partition to be equal to the worth of the coalition in the partition. This comes about from their "efficiency" axiom according to which the worth of the grand coalition is assumed to accrue as the total payoff of all players in any partition.

Apart from the δ-core, the classical α- and β- cores (Aumann, 1961) not only assume formation of the binary partition $\{S, N\backslash S\}$ subsequent to a deviation by coalition S, but also require the complementary coalition $N\backslash S$ to take actions that minimax or maximin the payoff of the deviating coalition S without regard to its own payoff.[1] There are also core concepts in which the partition that can be formed subsequent to a deviation is determined endogenously, but by imposing an exogenous selection criterion. These include the c-core in which the remaining players are assumed to form coalitions such that the payoff of the deviating coalition is minimized and the r-core in which the remaining players are assumed to form coalitions such that the sum of their payoffs is maximized.[2] Similarly, Huang and Sjöstörm (2003) and Kóczy (2007) introduce a core concept, also named the r-core or the recursive core, by (exogenously) imposing a consistency requirement on the partition that can be formed by the players outside the deviating coalition. In an important paper, Bloch and van den Nouweland (2013) evaluate the various core concepts in terms of axiomatic properties of their implicit expectation formation rules. In sum, all existing core concepts for partition function games make one or other *ad hoc* assumption concerning the partition that may be formed subsequent to a deviation. In contrast, we propose, in this paper, a core concept that does not rule out formation of any partition subsequent to a deviation except that the partition should include the deviating coalition.

[1] See Chander (2007) and Ray and Vohra (1997) for additional criticisms of the α- and β- cores.

[2] See Hafalir (2007) for formal definitions of both c- and r-cores.

Definition 3 The strong-core of a partition function game (N, v) is the set of all payoff vectors (x, N) such that for every deviating coalition S and every partition $P \ni S$ either $v(S; P) \leq \sum_{i \in S} x_i$ or $v(T; P) \leq \sum_{i \in T} x_i$ for some non-singleton coalition $T \in P$. That is, in every partition that could possibly be formed by a deviating coalition either the deviating coalition itself is worse-off or some non-singleton coalition is worse-off.

This definition of the strong-core is mathematically clear and precise and comparable to definitions of the γ- and δ- cores in that the strong-core does not assume formation of any particular partition subsequent to a deviation by a coalition. Also, it does not select among the partitions that could possibly form subsequent to a deviation by imposing an exogenous criterion. Furthermore, it is consistent with the traditional core in the sense that the strong-core reduces to the traditional core if the worth of every coalition is independent of the partition to which it belongs and the partition function is adequately represented by a characteristic function.

I now interpret and justify the strong-core. One interpretation is as follows: Suppose that a payoff vector (x, N) is under discussion of the players and must be collectively accepted or rejected. Now suppose that a coalition S thinks that it can do better than (x, N) provided that a particular partition $P \neq N, P \ni S$, is formed and some payoff vector $(y_1, \ldots, y_n)$ which is feasible for P is chosen. But since in partition function games, unlike characteristic function games, the payoff of coalition S depends on the entire coalition structure, it must convince all involved in the formation of partition P to agree to the alternative proposal. The question is: what are the *minimum* conditions that the alternative proposal $(y, P), P \neq N$, must fulfil for S to succeed in convincing all involved. A *necessary* condition for the alternative proposal to be acceptable to all involved is that no non-singleton coalition in the partition P should be worse-off.[3] Clearly, this is not a sufficient condition for acceptability of an alternative proposal. The strong-core payoff vectors rule out possibility of any alternative proposal that satisfies even this necessary condition for acceptability, and are thus strongly stable.

It is instructive to consider an alternative definition of the core: the core is the set of all payoff vectors (x, N) such that in every partition that could possibly be formed by a deviating

[3] This is conceptually analogous to the premise in Salant et al. (1983) that market structures in which some cartel (i.e. non-singleton coalition) is unprofitable would never occur in equilibrium with one difference, however, in that the status quo in Salant et al. is the Nash equilibrium payoff vector whereas here it is the grand coalition with a feasible payoff vector.

coalition at least one (singleton or non-singleton) coalition is worse-off. But this so-defined core is "too weak" a concept in the sense that it has no bite at least in the common class of games in which the grand coalition is the unique efficient coalition structure, i.e. $v(N;\{N\}) > \sum_{i=1}^{m} v(S_i;P)$ for every partition $P = \{S_1, \dots, S_m\} \neq \{N\}$. For this familiar class of games including the oligopoly games, the so-defined core consists of *all* payoff vectors (x, N).

Yet another definition of the core could be as the set of all payoff vectors (x, N) such that in every partition that can possibly be formed by a deviating coalition either the deviating coalition or some *singleton* coalition is worse-off relative to (x, N). But this definition of the core implies that the grand coalition may not be stable even if the so-defined core is nonempty. In fact, partitions, other than the grand coalition, such that *only* some singleton coalition is worse-off relative to each so-defined core payoff vector can be stable, because a singleton coalition alone cannot change a partition to which it belongs and achieve a higher payoff – it needs consent and help of at least one other coalition in the partition to effect any change in the partition, but there may be no such coalition in the partition. In contrast, a worse-off non-singleton coalition in a partition by itself can change the partition by just breaking apart into smaller coalitions. Whereas a singleton coalition cannot.

A further interpretation of the strong-core is as the set of all payoff vectors that are feasible for the grand coalition and cannot be "blocked" by any coalition by forming a partition in which at most singleton coalitions are worse-off. Blocking a feasible payoff vector in this set with a partition in which some non-singleton coalition is worse-off is not credible, since such a partition, as will be shown in Section 4, is not stable and is farsightedly dominated by each payoff vector in the set. This interpretation of the strong-core implicitly assumes that the deviating coalitions are not only farsighted but also conservative in the sense that a coalition does not deviate if the deviation may result in a "dominance chain" that terminates at a feasible payoff vector in which it is not better-off.

To further clarify the strong-core concept, consider the three-player game depicted in Fig. 1 below and in which the players' payoffs are derived from a symmetric Cournot oligopoly when each coalition in each partition distributes its payoff equally among its members. In this game, the payoff vector $(\frac{4}{3}, \frac{4}{3}, \frac{4}{3})$ is feasible for the grand coalition. We claim that this payoff vector belongs to the strong-core. This is because if coalition $\{3\}$ deviates and the resulting partition is $\{\{1,2\},\{3\}\}$, then the non-singleton coalition $\{1,2\}$ is worse-off, as $\frac{9}{8} + \frac{9}{8} < \frac{4}{3} + \frac{4}{3}$, and if the resulting partition is $\{\{1\},\{2\},\{3\}\}$, then the deviating coalition $\{3\}$ itself is worse-off, as $1 < \frac{4}{3}$. Thus, if $\{3\}$ deviates, then in *all* partitions that may possibly form subsequent to its deviation and that include the deviating coalition, $\{3\}$, either a non-singleton coalition, $\{1,2\}$, is worse-off or the deviating coalition, $\{3\}$, itself is worse-off. It is easy to verify similarly that this is also the case for deviations by other coalitions.

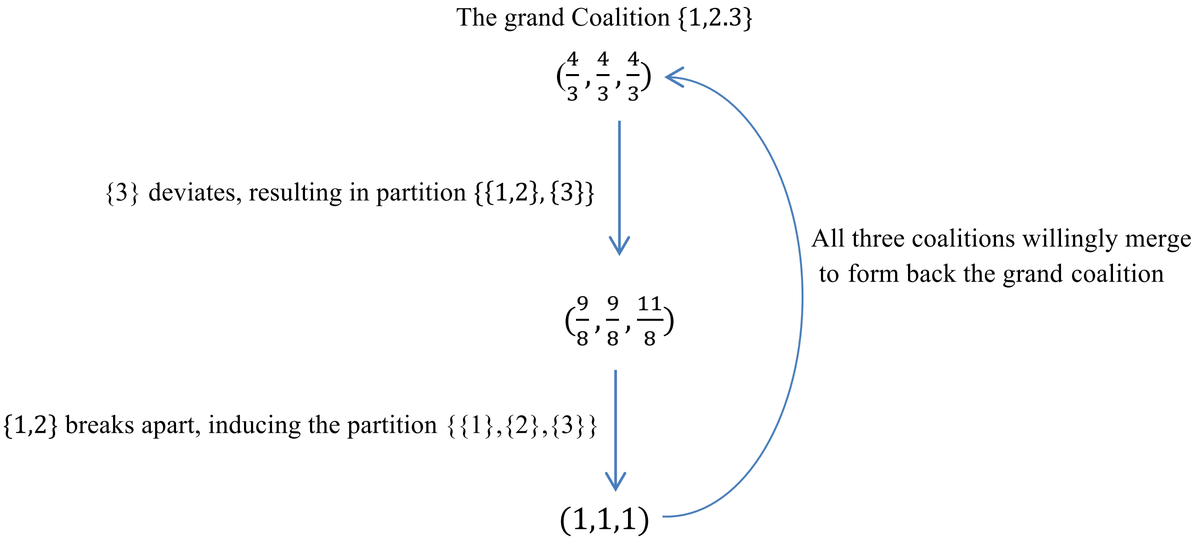

Figure1

Now notice that if $\{3\}$ deviates from the grand coalition with payoff vector $(\frac{4}{3},\frac{4}{3},\frac{4}{3})$, it cannot be sure that the resulting partition will indeed be $\{\{1,2\},\{3\}\}$ and it will get the "free rider's" payoff of $\frac{11}{8} > \frac{4}{3}$. This is because the partition $\{\{1,2\},\{3\}\}$ is not stable, since the non-singleton coalition $\{1,2\}$ is worse-off compared to its payoff in the grand coalition with payoff vector $(\frac{4}{3},\frac{4}{3},\frac{4}{3})$. In fact, the worse-off coalition $\{1,2\}$ has incentive and can successfully deter the deviation by $\{3\}$ by first breaking apart, leading to payoffs $(1,1,1)$, and then proposing to form back the grand coalition with payoffs $\left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right) > (1,1,1)$. Notice that once coalition $\{1,2\}$ breaks apart, the deviating coalition $\{3\}$ will agree to form back the grand coalition with payoff vector $(\frac{4}{3},\frac{4}{3},\frac{4}{3}) > (1,1,1)$. Thus, the worse-off non-singleton coalition $\{1,2\}$ has incentive to break apart, as that would lead back to formation of the grand coalition and higher payoff for the coalition then if it does not break apart, since $(\frac{4}{3},\frac{4}{3}) >$

$\left(\frac{9}{8}, \frac{9}{8}\right)$. The immediate loss in the payoff of coalition {1,2} upon breaking apart is only temporary.

The above discussion and illustration of the strong-core also highlights that the strong core implicitly assumes that the players are farsighted and conservative. This will be made more clear and precise in Section 4.

2.2 *Some important inclusion relationships*

The γ-core proposed as an improvement over the classical α- and β- cores has been applied to a wide range of economic models and found useful for analysing situations involving externalities such as climate change. See the list of references at the end of the paper. Thus, a relevant question regarding the strong-core is whether it is consistent with the γ-core.

Proposition 1 The strong-core of a partition function game (N, v) is a stronger concept than the γ-core in the sense that strong-core $\subset$ γ-core in general. But the strong-core is not generally equal to the γ-core.

Proof: Let (x, N) be a strong-core payoff vector. Since in every partition $\{S, [N\backslash S]\} \neq [N]$ at most coalition S is a non-singleton, it follows from definition of the strong-core payoff vectors that $\sum_{i\in S} x_i \geq v(S; \{S, [N\backslash S]\}$ for every non-singleton coalition S. Furthermore, this inequality also holds for every singleton coalition S, since if the deviating coalition S is a singleton then the partition $\{S, [N\backslash S]\}$ contains no singleton coalition and, therefore, the deviating coalition S itself must be worse-off. Thus, (x, N) is also a γ-core payoff vector. But two are not generally equal, as the following example shows.

Let $= \{1,2, \ldots, 5\}$, $v(N; N) = 13$, $v(S; \{S, [N\backslash S]\}) = 2.4|S|$, $v(S; \{S; N\backslash S\}) = 2.6|S|$ for $|S| < 4$, $v(S; \{S; N\backslash S\}) = 2.4|S|$ for $|S| = 4$, for each partition $P = \{\{ij\}, \{kl\}, \{m\}\}$, $v(\{ij\}; P) = v(\{kl\}; P\} = 6$ and $v(\{m\}; P) = 1$, for each partition $P = \{\{i\}, \{j\}, \{k\}, \{lm\}\}$, $v(\{i\}; P) = 1$, and for each partition $P = \{\{i\}, \{j\}, \{klm\}\}$, $v(\{i\}; P) = 1$.

In this game, the feasible payoff vector $(x_1, x_2, \ldots, x_5) = (2.6, 2.6, \ldots, 2.6)$ belongs to the γ-core and thus the γ-core is nonempty. But the strong-core is empty. This is seen as follows: A feasible payoff vector $(x_1, x_2, \ldots, x_5)$, by definition, belongs to the strong-core only if $\sum_{i\in N} x_i = 13$, $x_i \geq 2.4$, $i = 1,2, \ldots, 5$, and at least for the partition $P = \{\{12\}, \{34\}, \{5\}\}$, either $v(\{12\}; P) \leq x_1 + x_2$ or $v(\{34\}; P) \leq x_3 + x_4$. But there can be no such feasible vector, since $x_i \geq 2.4$, $i = 1,2, \ldots, 5$ and, therefore, $x_1 + x_2 = 13 - x_3 - x_4 - x_5 \leq 5.8 < v(\{12\}; P)$ and $x_3 + x_4 = 13 - x_1 - x_2 - x_5 \leq 5.8 < v(\{34\}; P)$. Hence, the strong-core is empty, but the γ-core is not. Thus, the strong-core is strictly smaller than the γ-core. This is because the γ-core is determined *only* by the payoffs $v(S; \{S, [N \backslash S]\}, S \subset N$, and, unlike the strong-core, independent of the payoffs $v(S; P)$, $P = \{\{ij\}, \{kl\}, \{m\}\}, S \in P$. Thus, externalities from coalition formation play a greater role in the determination of the strong-core. ■

Intuitively, the strong-core is smaller than the γ-core because, unlike the γ-core, the strong-core, by definition, does not rule out formation of any partition subsequent to a deviation. Proposition 1 can be viewed as an extension of a previous consistency property of alternative core concepts in that it is known (Chander, 2016: Proposition 1) that in general strategic games, γ-core $\subset$ β-core $\subset$ α-core. Proposition 1 implies that for general strategic games, strong-core $\subset$ γ-core $\subset$ β-core $\subset$ α-core. This is a nice property to have, since it means that applications of the strong-core to strategic games that can be converted into partition function games, are not inconsistent with applications of α-, β-, and γ- cores. As will be shown, the strong-core of an oligopoly is strictly smaller than the γ-core and thus is also smaller than the α- and β- cores.

Similarly, on the one hand, the strong-core is a stronger concept than the δ-core, since the δ-core, by definition, rules out formation of all but binary partitions subsequent to a deviation. But, on the other hand, the strong-core is a weaker concept because it, unlike the δ-core, requires only one, not both (as noted in the paragraph following the definition of the δ-core) coalitions in every binary partition to be worse-off. For this reason, the strong-core and the δ-core are not generally comparable except in some special cases of interest, as shown below.

Yi (1997) and Maskin (2003) note that in most applications the partition function games can be classified into two classes, namely, as games with "positive" or "negative" externalities.

A partition function game (N, v) has *positive* (*negative*) externalities if for every partition $P = \{S_1, \ldots, S_m\}$ and $S_i, S_j \in P$, we have $v(S_k; P \backslash \{S_i, S_j\} \cup \{S_i \cup S_j\}) \geq (\leq) v(S_k; P)$ for each $S_k \in P, k \neq i, j$.

In words, a partition function game has positive (negative) externalities if a merger between any two coalitions in a partition increases (decreases) the worth of every other coalition in the partition.[4] It is known that for games with positive externalities, δ-core $\subset$ γ-core. I now establish an additional inclusion relationship.

Proposition 2 (a) For partition function games with positive externalities, δ-core $\subset$ strong-core $\subset$ γ-core, and (b) for games with negative externalities, strong-core = γ-core $\subset$ δ-core.

Proof: (a) First, suppose contrary to the assertion that in a game with positive externalities a δ-core payoff vector $(x_1, \dots, x_n)$ does not belong to the strong-core. Since $(x_1, \dots, x_n)$ belongs to the δ-core, for every coalition $S \subset N$ and partition $\{S, N\backslash S\}$, $\sum_{i \in S} x_i \geq v(S; \{S, N\backslash S\}) \geq v(S; \{S, [N\backslash S]\})$, since externalities are positive. In particular, for $S = \{i\}$, $x_i \geq v(i; [N]), i = 1, \dots, n$. Furthermore, since $(x_1, \dots, x_n)$, by supposition, does not belong to the strong-core, there must exist a partition $P = \{S_1, \dots, S_m\} \neq [N]$ such that $v(S_i; P) > \sum_{j \in S_i} x_j$ for all $S_i \in P$ with $|S_i| > 1$. Then, since externalities are positive, $v(S_i; P') > \sum_{j \in S_i} x_j$, where $P' = \{S_i, N\backslash S_i\}$. But this contradicts that $(x_1, \dots, x_n)$ belongs to the δ-core. Hence our supposition is wrong and, therefore, every δ-core payoff vector $(x_1, \dots, x_n)$ belongs to the strong-core. This proves that δ-core $\subset$ strong-core.

Second, if $(x_1, \dots, x_n)$ belongs to the strong-core, then, by definition, $\sum_{i \in S} x_i \geq v(S; [N\backslash S])$ for all non-singleton coalitions S and for the partition $P = [N]$, $x_i \geq v(i; [N]), i = 1, \dots, n$. Thus, every strong-core payoff vector $(x_1, \dots, x_n)$ belongs to the γ-core. This proves strong-core $\subset$ γ-core.

(b) First, let $(x_1, \dots, x_n)$ be a γ-core payoff vector of a partition function game (N, v) with negative externalities. We claim that $(x_1, \dots, x_n)$ also belongs to the strong-core. Suppose not. Since $(x_1, \dots, x_n)$ belongs to the γ-core, for every partition $\{S, [N\backslash S]\}, S \subset N$, $\sum_{i \in S} x_i \geq v(S; \{S, [N\backslash S]\})$ and $x_i \geq v(i; [N]), i = 1, \dots, n$. Then, since $(x_1, \dots, x_n)$, by supposition, does not belong to the strong-core, there must be a partition $P = \{S_1, \dots, S_m\} \neq [N]$ such that $v(S_i; P) > \sum_{j \in S_i} x_j$ for all $S_i \in P$ with $|S_i| > 1$. Let $P' = \{S_i, [N\backslash S_i]\}$ denote the partition in which all but coalition S_i is a singleton. Then, since the game (N, v) has negative externalities, $v(S_i; \{S_i, [N\backslash S_i]\}) \geq v(S_i; P) > \sum_{j \in S_i} x_j$. But this contradicts that $(x_1, \dots, x_n)$ is a γ-core payoff vector. Hence, our supposition is wrong and each γ-core payoff vector

[4] See Yi (1997), Maskin (2003), and Hafalir (2007) among others for the definition.

$(x_1, \dots, x_n)$ also belongs to the strong-core. This proves that γ-core $\subset$ strong-core. Since, as Proposition 1 shows, strong-core $\subset$ γ-core in general. Therefore, for games with negative externalities, strong-core = γ-core.

Second, suppose contrary to the assertion that for a game with negative externalities a strong payoff vector $(x_1, \dots, x_n)$ does not belong to the δ-core. Then, we must have $\sum_{i \in S} x_i < v(S; \{S, N \backslash S\})$ for some $S \subset N$. Since externalities are negative, this implies $\sum_{i \in S} x_i < v(S; \{S, [N \backslash S]\})$ for some $S \subset N$. But this contradicts that $(x_1, \dots, x_n)$ belongs to the strong-core. Thus our supposition is wrong and every strong-core payoff vector $(x_1, \dots, x_n)$ also belongs to the δ-core. This proves strong-core $\subset$ δ-core. ■

Examples can be constructed to show that both inclusions in part (a) of Proposition 2 are strict in the same game, i.e., there are games with positive externalities in which the strong-core is not equal to either the γ-core or the δ-core, but "sits" nicely between them.

2.3 *An additional property of the strong-core*

I now establish an additional property of the strong-core by restricting to another special class of partition function games.

Definition 4 A partition function game (N, v) is partially superadditive if for each partition $P = \{S_1, \dots, S_m\}$ with $|S_i| \geq 2, i = 1, \dots, k$, and $|S_j| = 1, j = k+1, \dots, m, k \leq m$, $\sum_{i=1}^{k} v(S_i; P) \leq v(S; P')$ where $P' = P \backslash \{S_1, \dots, S_k\} \cup \{\cup_{i=1}^{k} S_i\}$.

Partial superadditivity, as the term suggests, is weaker than the familiar notion of superadditivity, which requires that combining any number of *arbitrary* coalitions increases their worth.[5] In contrast, partial superadditivity requires that combining only *all non-singleton* coalitions increases their worth. Thus, partial superadditivity is weaker than superaddivity.

Proposition 3 Let (N, v) be a partially superadditive partition function game. Then the strong-core is equal to the γ-core.

[5] See Hafalir (2007) for a formal definition. However, Hafalir uses the term "fully cohesive" in place of "superadditive".

Proof: The strong-core, by definition, is a subset of the γ-core. Formally, if $(x_1, \dots, x_n)$ belongs to the strong-core, then, by definition, $\sum_{i\in S} x_i \geq v(S; [N\backslash S])$ for all non-singleton coalitions S and $x_i \geq v(i; [N]), i = 1,2, \dots, n$. Therefore, $(x_1, \dots, x_n)$ also belongs to the γ-core. Thus, we only need to prove that each γ-core payoff vector also belongs to the strong-core.

Let $(x_1, \dots, x_n)$ be a γ-core payoff vector and let $P = \{S_1, \dots, S_m\}$ be a partition of N. If $P = \{S_1, \dots, S_m\} \neq [N]$, then let $|S_i| > 1$, for $i = 1, \dots, k$ and $|S_j| = 1$ for $j = k + 1, \dots, m, k \leq m$, and $S = \cup_{i=1}^{k} S_i$. Since v is partially superadditive, $\sum_{i=1}^{k} v(S_i; P) \leq v(S; P')$ where $P' = P\backslash\{S_1, \dots, S_k\} \cup \{S\}$. Clearly, $P' = \{S, [N\backslash S]\}$. Since $(x_1, \dots, x_n)$ is a γ-core payoff vector, $\sum_{i\in S} x_i \geq v(S; \{S, [N\backslash S]\}) = v(S; P') \geq \sum_{i=1}^{k} v(S_i; P)$. This inequality can be rewritten as $\sum_{i=1}^{k} \sum_{j\in S_i} x_j \geq \sum_{i=1}^{k} v(S_i; P)$ and, therefore, $\sum_{j\in S_i} x_j \geq v(S_i; P)$ for at least one $S_i \in \{S_1, \dots, S_k\} \subset P$ with $s_i > 1$. If $P = [N]$, then since $(x_1, \dots, x_n)$ is a γ-core payoff vector, $x_i \geq v(i; \{i, [N\backslash i]\}) = v(i; [N])$. This proves that $(x_1, \dots, x_n)$ belongs to the strong-core. ■

Partial superadditivity is trivially satisfied in all three-player partition function games. It is also satisfied by those four-player partition function games, including the oligopoly games, in which the grand coalition is efficient. I apply Proposition 3 below to an oligopoly game in which firms have capacity constraints to show that the game is partially superadditive and, therefore, the strong-core is equal to the γ-core which is known to be nonempty (Lardon, 2012).[6]

2.4 *A class of games with nonempty strong-cores*

A number of studies on coalition formation have focused on symmetric partition function games in which larger coalitions in each partition have lower per-member payoffs (see e.g. Ray and Vohra, 1997, Funaki and Yamato, 1999, and Chander, 2007). It is known (Chander, 2016) that these games admit nonempty γ-cores. But the γ-cores of these games are not equal to the strong-cores as these games are not partially superadditive and strong-cores, as shown,

[6] Another application of Proposition 3 is that the games in Chander and Tulkens (1997) and Helm (2001) admit nonempty strong-cores if the 'damage functions' are linear, since then these games are partially superadditive and the γ-cores of these games are known to be nonempty.

are generally smaller. I now prove a stronger result in that I show that these games actually admit nonempty strong-cores.

Proposition 4 Let (N, v) be a symmetric partition function game such that for every partition $P = \{S_1, \dots, S_m\}$, $v(S_i; P)/|S_i| < (=) v(S_j; P)/|S_j|$ if $|S_i| > (=) |S_j|, i, j \in \{1, \dots, m\}$ and $v(N; N) > \sum_{S_i \in P} v(S_i; P)$. Then, (N, v) admits a nonempty strong-core.

Proof: Let $(x_1, \dots, x_n)$ be the feasible payoff vector with equal shares, i.e., $\sum_{i \in N} x_i = v(N; N)$ and $x_i = x_j, i, j \in N$. We claim that $(x_1, \dots, x_n)$ is a strong-core payoff vector.

Let $P = \{S_1, \dots, S_m\} \neq N$ be some partition of N. If $P = [N]$, then $x_i \geq v(i; [N])$ for all $\{i\} \in [N]$, since $v(N; N) > \sum_{i \in N} v(i; [N])$, $\sum_{i \in N} x_i = v(N; N)$, $v(i; [N]) = v(j; [N])$, and $x_i = x_j$ for all $i, j \in N$. If $P \neq [N]$, then the number of coalitions in the partition is $m \geq 2, m < n$. Without loss of generality assume that $|S_1| \geq |S_2| \geq \dots \geq |S_m|$. Thus, $n > m \geq 2$ and $\sum_{i=1}^{m} v(S_i; P) < v(N; N) = \sum_{i \in N} x_i$, as hypothesized. This inequality implies $v(S_1; P) < \sum_{i \in S_1} x_i$, since $v(S_1; P)/|S_1| \leq v(S_j; P)/|S_j|$ for all $S_j \in P$ and $x_i = x_j, i, j \in N$. Since $n \geq 3$ and $P \neq [N], N$, we must have $|S_1| \geq 2$. This proves that each partition $P \neq [N]$, includes at least one non-singleton coalition which is worse-off relative to the feasible payoff vector with equal shares $(x_1, \dots, x_n)$ and $x_i \geq v(i; [N]), i = 1, \dots, n$. By Definition 3, $(x_1, \dots, x_n)$ belongs to the strong-core. ∎

3. The strong-core of an oligopoly

In this section, I first define an oligopoly as a strategic game. Then, as in the approach pioneered by Ichiishi (1981) and Zhao (1996), I convert the strategic oligopoly game into a partition function game by proving existence of a unique Nash equilibrium for each induced strategic game in which each coalition in each partition acts as one single player and the payoff to each coalition in a partition is the sum of profits that accrue to each of its members. As will be seen below, this conversion is based on outputs, prices and thus profits that are endogenously determined in exactly the same way as in Salant et al. (1983).

The set of oligopolistic firms is $N = \{1, \dots, n\}$. Let $p(q)$ denote the inverse demand function faced by these firms, where q is the total demand. I assume that the inverse demand function is differentiable and strictly decreasing and concave, i.e., $p'(q) < 0$ and $p''(q) \leq$

0.[7] These assumptions imply that the revenue function $p(q)q_i$ of each firm i is concave in $q_1, \dots, q_n$, i.e. the marginal revenue $p(q) + p'(q)q_i$ of each firm i is non-increasing in the output of other firms, since $p'(q) + p''(q)q_i \leq 0$ for each fixed $q_i \geq 0$, as well as in its own output, given the output of other firms, since $2p'(q) + p''(q)q_i \leq 0$ for all fixed $q_j \geq 0, j \neq i$.

The cost function of firm i is $c_i(q_i)$ with $c_i(0) = 0$. I assume that the cost function of each firm is differentiable, strictly increasing and strictly convex, i.e., $c_i'(0) = 0$, $c_i'(q_i) > 0, q_i > 0$, and $c_i''(q_i) > 0, q_i \geq 0$. Assuming a strictly convex cost function implies that a coalition of two or more firms can produce the same output at a lower cost than a single, standalone firm. Thus, there is an additional incentive for the firms to cooperate. For the time being I do not impose any capacity constraints on the outputs of the firms and assume that the firms can expand their production capacity as and when necessary. This leads to a long-run equilibrium analysis of an oligopoly.

The profit function of each firm i is $\pi_i(q_1, \dots, q_n) = p(q)q_i - c_i(q_i)$, where $q = \sum_{j \in N} q_j$. In order to avoid corner solutions, I assume that there exists an upper bound q^0 such that $p(q^0) + p\,'(q^0)q^0 - c_i'(q^0) < 0$ for each firm i. Since both $p(q)$ and $p'(q)$ are non-increasing functions of q, this assumption implies that a standalone profit maximizing firm or a firm as a member of a profit maximizing coalition will never produce an output larger than q^0 even if it has the capacity to do so. We assume further that $p((n-1)q^0) > 0$. Since $c_i'(0) = 0$, this assumption implies that a standalone profit maximizing firm or a firm as a member of a profit maximizing coalition will always produce a positive amount irrespective of the output of other profit maximizing firms or coalitions.

3.1 *The oligopoly game*

Let $A_i = [0, q^0], A = A_1 \times \cdots \times A_n$, and $\pi = (\pi_1, \dots, \pi_n)$. I shall refer to the strategic game (N, A, π) as the *oligopoly game*. Clearly, each strategy set A_i is compact and convex and each π_i is concave and continuous in $q_1, \dots, q_n$.

Lemma 5 The oligopoly game (N, A, π) admits a unique Nash equilibrium $(\bar{q}_1, \dots, \bar{q}_n)$.

[7] It is worth noting that a linear demand function $p(q) = a - bq$ satisfies these assumptions.

Proof: See the appendix to the paper. ■

Let (N^P, A^P, π^P) denote the induced game when the firms form a partition $P = \{S_1, \dots, S_m\}$ and each coalition $S_i, i = 1, \dots, m,$ in the partition acts as a single player. Since each $\pi_i(q_1, \dots, q_n)$ is concave and continuous in $q_1, \dots, q_n$, the payoff function $\pi^P_{S_i}(q_1, \dots, q_n) \equiv \sum_{j \in S_i} \pi_j(q_1, \dots, q_n))$ of coalition S_i is also concave and continuous in $q_1, \dots, q_n$. Moreover, the strategy set $\times_{j \in S_i} A_j$ of coalition S_i is compact and convex. Therefore, as in Lemma 5, the induced game (N^P, A^P, π^P) also admits a unique Nash equilibrium $(\bar{q}^P_1, \dots, \bar{q}^P_n)$. Let $v(S_i; P)$ be equal to the Nash equilibrium payoff of coalition S_i in the induced game, i.e. $v(S_i; P) = \sum_{j \in S_i} \pi_j\, (\bar{q}^P_1, \dots, \bar{q}^P_n)$. Then, (N, v) is the partition function game form of the oligopoly game (N, A, π). Clearly, the grand coalition in this partition function game is efficient, since the grand coalition can choose at least the same strategies as the coalitions in any partition.

Since all firms face the same demand function, the firms are identical if their cost functions are equal, i.e. $c_i(.) = c_j(.), i, j \in N$.

Proposition 6 If all firms are identical, then the partition function game form (N, v) of the oligopoly game (N, A, π) is symmetric and admits a nonempty strong-core.

Proof: For each partition $P = \{S_1, \dots, S_m\}$, let $(\bar{q}^P_1, \dots, \bar{q}^P_n)$ denote the unique Nash equilibrium of the induced game (N^P, A^P, π^P). Then, by the first order conditions, $c'_i(\bar{q}^P_i) = p(\sum_{i \in N} \bar{q}^P_i) + (\sum_{j \in S_k} \bar{q}^P_j)\, p'(\sum_{i \in N} \bar{q}^P_i)$, $i \in S_k$. Since the firms are identical $\bar{q}^P_i = \bar{q}^P_j, i, j \in S_k$. Since each c_i is strictly convex, these equalities imply that $\bar{q}^P_j > (=) \bar{q}^P_i$ if $j \in S_k \in P, i \in S_r \in P$ and $|S_k| < (=) |S_r|$. That is if the firms are identical and form cartels, the output of each firm in a larger cartel is lower. Thus, since all firms are identical and face the same prices, $v(S_k; P)/|S_k| > (=) v(S_r; P)/|S_r|$ if $|S_k| < (=) |S_r|, k, r \in \{1, \dots, m\}$. The proof now follows from Proposition 3. ■

Proposition 6 is more general than previous similar results in at least three respects. First, it holds for an oligopoly with any finite number of firms, second it holds for more general demand and cost functions,[8] and third, it proves the existence of a nonempty strong-core,

[8] Most studies assume specific demand and cost functions.

which is generally a subset of the γ-core.[9] The following example which belongs to the class of oligopoly games in Proposition 6 confirms that in many cases it is actually a strict subset.

Proposition 6 means that in a symmetric oligopoly at least one cartel (i.e. a non-singleton coalition) in any market structure is worse-off then under monopoly with equal sharing of its profits among its members. Thus, any break up of monopoly would be opposed by at least one group of firms.

Example 1 Let $N = \{1,2,3,4,5\}$, $p(q) = 1 - q$ and $c_i(q_i) = \frac{1}{2}q_i^2, i \in N$.

We first show that the example does not satisfy the sufficient condition for equality of the strong core and the γ-core, i.e., partition function game representation (N, v) of this oligopoly is not partially superadditive. Let $P = \{\{1\},\{2,3\},\{4,5\}\}$ and $H = \{\{1\},\{2,3,4,5\}\}$. We claim that $v(\{2,3,4,5\}; H) < v(\{2,3\}; P) + v(\{4,5\}; P)$. It is easily seen that $\bar{q}_1^P = \frac{3}{17}, \bar{q}_2^P = \bar{q}_3^P = \bar{q}_4^P = \bar{q}_5^P = \frac{2}{17}$ and, thus, $v(\{2,3\}; P) = v(\{4,5\}; P) = 5\,(\frac{2}{17})^2$. Similarly, $\bar{q}_1^H = \frac{5}{23}, \bar{q}_2^H = \bar{q}_3^H = \bar{q}_4^H = \bar{q}_5^H = \frac{2}{23}$ and $v(\{2,3,4,5\}; H) = 18(\frac{2}{23})^2 < v(\{2,3\}; P) + v(\{3,4\}; P) = 5\,(\frac{2}{17})^2 + 5\,(\frac{2}{17})^2$.

It is noteworthy that $\bar{q}_1^H = \frac{5}{23} > \bar{q}_1^P = \frac{3}{17}$, but $\bar{q}_2^H = \bar{q}_3^H = \bar{q}_4^H = \bar{q}_5^H = \frac{2}{23} < \bar{q}_1^P = \frac{3}{17}, \bar{q}_2^P = \bar{q}_3^P = \bar{q}_4^P = \bar{q}_5^P = \frac{2}{17}$, that is if the two cartels $\{2,3\}$ and $\{4,5\}$ merge then each firm in these cartels produces a smaller output, but the outside firm $\{1\}$ produces a larger output. This is because every firm in the merged cartels must take into account the effect of its production not only on its own profit, but also on the profits of all other firms in the merged cartels. But a decrease in the outputs of the firms in the merged cartels makes it profitable for the oligopolistic firm outside the cartels to raise its output.

Since partial superadditivity is a sufficient, not a necessary, condition for equality of the strong-core and the γ-core, it remains to be verified that the strong-core of this oligopoly is indeed smaller. To that end, we note that $v(N; \{N\}) = \frac{5}{22}$. Let $x_1 = v(N; \{N\}) -$

[9]Rajan (1989) proves existence of a nonempty γ-core for a symmetric oligopoly with only three firms. Lardon (2012) proves existence of a nonempty γ-core for an oligopoly with exogenously fixed production capacities. Radner (2001) and Zhao (1999) prove non-emptiness of the traditional α- and β- cores, which are generally larger than even the γ-core and, therefore, the strong core.

$v(\{2,3,4,5\}; H) = \frac{5}{22} - 18(\frac{2}{23})^2$ and $x_2 = x_3 = x_4 = x_5 = \frac{1}{4} v(\{2,3,4,5\}; H) = \frac{18}{4} (\frac{2}{23})^2$. Then the feasible payoff vector $(x_1, \dots, x_5)$ belongs to the γ-core, since the game is symmetric and $\frac{1}{4} v(\{2,3,4,5\}; H) > \frac{1}{3} v(\{3,4,5\}; \{\{1\}, \{2\}, \{3,4,5\}\}) > \frac{1}{2} v(\{4,5\}; \{\{1\}, \{2\}, \{3\}, \{4,5\}\}) > v(\{i\}; [N])$ and $x_1 > \frac{1}{4} v(\{2,3,4,5\}; H)$. But the feasible payoff vector $(x_1, \dots, x_5)$ does not belong to the strong-core, since $x_2 + x_3 = x_4 + x_5 = \frac{18}{2} (\frac{2}{23})^2 < v(\{2,3\}; P) = v(\{4,5\}; P) = 5 (\frac{2}{17})^2$.

The example may look unnecessarily complicated, but this is because in games in which the grand coalition is efficient there have to be at least five players for the strong-core to be strictly smaller than the γ-core. It is known that oligopoly games exhibit positive externalities and therefore the δ-core is smaller than the γ-core. As shown in Proposition 2, it is in fact smaller than even the strong-core. Indeed, Rajan (1989: Theorem 1) shows that the δ-core of a symmetric oligopoly with just three firms is empty but the γ-core is not (Rajan, 1989: Theorem 2) and, therefore, the strong core is also not empty (by Proposition 3), as there are only three firms and thus the game is partially superadditive and the strong-core is equal to the γ-core.

3.2 *An oligopoly with capacity constraints*

A number of studies (e.g. Radner, 2001,Yong, 2004, Lardon, 2012) focus on oligopolies with firms with arbitrary capacity constraints. In the same vein, I now prove existence of a nonempty strong-core for a specific set of capacity constraints. In particular, I assume that each firm i in an oligopoly faces a capacity constraint $k_i = \bar{q}_i, i = 1, \dots, n$, where $\bar{q}_i$ is its Nash equilibrium output in the oligopoly game (N, A, π). Such capacity constraints seem natural, since if an antitrust policy is in place, the firms would produce according to their Nash equilibrium strategies and build production capacities accordingly. The question is whether with these capacity constraints also oligopolistic firms will merge to a monopoly if the antitrust policy is abolished.

Let $\bar{A}_i = [0, \bar{q}_i], i \in N$, and $\bar{A} = \bar{A}_1 \times \dots \times \bar{A}_n$.[10] I shall refer to the strategic game $(N, \bar{A}, \pi)$ as the *constrained* oligopoly game. Let $(N^P, \bar{A}^P, \pi^P)$ denote the induced game when the

[10] This means implicitly that the production capacities of the firms cannot be transferred or pooled and cartel formation does not confer any advantage in terms of size on the firms in the cartel. The same is also true in

capacity constrained firms form a partition $P = \{S_1, \dots, S_m\}$ and each coalition $S_i, i = 1, \dots, m$, in the partition acts as a single firm. Clearly, the strategy set $\times_{j \in S_i} \bar{A}_j$ of coalition S_i is compact and convex and, by the same arguments as in Lemma 5, each induced game $(N^P, \bar{A}^P, \pi^P)$ admits a unique Nash equilibrium $(\bar{\bar{q}}_1^P, \dots, \bar{\bar{q}}_n^P)$. In particular, the Nash equilibrium $(\bar{q}_1, \dots, \bar{q}_n)$ of the original oligopoly game (N, A, π) is also the unique Nash equilibrium of the constrained oligopoly game $(N, \bar{A}, \pi)$.

Proposition 7 For each partition $P = \{S, [N \backslash S]\}, S \subset N$, let $(\bar{\bar{q}}_1^P, \dots, \bar{\bar{q}}_n^P)$ denote the unique Nash equilibrium of the induced constrained game $(N^P, \bar{A}^P, \pi^P)$. Then, (i) $\bar{\bar{q}}_j^P = \bar{q}_j$ for each $j \in N \backslash S$ and (ii) $\bar{\bar{q}}^P \equiv \sum_{i \in N} \bar{\bar{q}}_i^P < \sum_{i \in N} \bar{q}_i \equiv \bar{q}$ That is if a cartel forms then the output of each independent firm is equal to its full capacity output, but the total industry output is lower compared to the Nash equilibrium output.

Proof: (i) By definition of the induced constrained game, $\bar{\bar{q}}_i^P \leq \bar{q}_i, i \in N$, and, thus, $\bar{\bar{q}}^P \leq \bar{q}$. Suppose contrary to the assertion that $\bar{\bar{q}}_j^P < \bar{q}_j$ for some $j \in N \backslash S$. Then, by the first order conditions for a Nash equilibrium, $c_j'(\bar{q}_j) = p(\bar{q}) + \bar{q}_j p'(\bar{q}) < p(\bar{q}) + \bar{\bar{q}}_j^P p'(\bar{q}) \leq p(\bar{\bar{q}}^P) + \bar{\bar{q}}_j^P p'(\bar{\bar{q}}^P) = c_j'(\bar{\bar{q}}_j^P) \Rightarrow \bar{q}_j < \bar{\bar{q}}_j^P$, since c_j is strictly convex. But this contradicts our supposition that $\bar{\bar{q}}_j^P < \bar{q}_j$. Therefore, $\bar{\bar{q}}_j^P = \bar{q}_j$ for all $j \in N \backslash S$.

(ii) Suppose contrary to the assertion that $\bar{\bar{q}}^P = \bar{q}$. Then, $\bar{\bar{q}}_j^P = \bar{q}_j$ for all $j \in N$ and for each $i \in S$, $c_i'(\bar{\bar{q}}_i^P) = (\sum_{j \in S} \bar{\bar{q}}_j^P)\, p'(\bar{\bar{q}}^P) + p(\bar{\bar{q}}^P) = (\sum_{i \in S} \bar{q}_i) p'(\bar{q}) + p(\bar{q}) < \bar{q}_i p'(\bar{q}) + p(\bar{q}) = c_i'(\bar{q}_i) \Rightarrow \bar{q}_i > \bar{\bar{q}}_i^P$, since c_i is strictly convex. But this contradicts our supposition that $\bar{\bar{q}}^P = \bar{q}$ and $\bar{\bar{q}}_j^P = \bar{q}_j, j \in N$. Therefore, $\bar{\bar{q}}^P < \bar{q}$. ■

Proposition 7 implies that if a cartel forms, then in the resulting equilibrium the total output is lower compared to the Nash equilibrium output and thus the prices are higher and the firms outside the cartel are better-off even though they cannot increase their outputs because of the capacity constraints; the firms in the cartel are not necessarily better-off because, on the one hand, they could be better-off because the prices are higher but, on the other hand, they could be worse-off because of their total output is lower. Thus, despite

models which allow pooling of production capacities, but assume firms with identical technologies and constant returns to scale.

capacity constraints, Proposition7 is consistent with the analysis in Salant et al. (1983) as well as with Segal (1999) who shows that cartel formation may confer positive externality on outside firms.[11]

The partition function game form of an oligopoly, as Example 1 shows, is typically not partially superadditive. But it is, if the firms face capacity constraints equal to their Nash equilibrium outputs, as the next proposition shows.

Proposition 8 The partition function game representation of the constrained oligopoly game $(N, \bar{A}, \pi)$ is partially superadditive.

Proof: As in Lemma 5, for each partition $P = \{S_1, \dots, S_m\}$, the induced game $(N^P, \bar{A}^P, \pi^P)$ admits a unique Nash equilibrium. Let $(\bar{\bar{q}}_1^P, \dots, \bar{\bar{q}}_n^P)$ denote the unique Nash equilibrium. Then, by definition of the constrained oligopoly game, $\bar{\bar{q}}^P \equiv \sum_{i \in N} \bar{\bar{q}}_i^P \leq \bar{q} = \sum_{i \in N} \bar{q}_i = \sum_{i \in N} k_i$. We claim that, as in Proposition 7, $\bar{\bar{q}}_j^P = \bar{q}_j = k_i$ for each *singleton* coalition $\{j\}$ in partition P. Suppose contrary to the claim that $\bar{\bar{q}}_j^P < \bar{q}_j$ for some singleton coalition $\{j\} \in P$. Then, $c_j'(\bar{\bar{q}}_j^P) = p(\bar{\bar{q}}^P) + \bar{\bar{q}}_j^P p'(\bar{\bar{q}}^P) \geq p(\bar{q}) + \bar{\bar{q}}_j^P p'(\bar{q}) > p(\bar{q}) + \bar{q}_j p'(\bar{q}) = c_j'(\bar{q}_j) \Rightarrow \bar{q}_j < \bar{\bar{q}}_j^P$, since c_j is strictly convex. But this is a contradiction. Thus, $\bar{\bar{q}}_j^P = \bar{q}_j = k_j$ for each singleton firm j in P.

Let $\bar{v}(S_i, P) = \sum_{j \in S_i} \pi_j (\bar{\bar{q}}_1^P, \dots, \bar{\bar{q}}_n^P)$. Without loss of generality assume that in partition $P = \{S_1, \dots, S_m\}$, $|S_i| > 1, i = 1, \dots, r$, and $|S_j| = 1, j = r+1, \dots, m$. Let $S = \cup_{i=1}^r S_i$, $H = P \backslash \{S_1, \dots, S_r\} \cup \{S\} = \{S, [N \backslash S]\}$ and let $(\bar{\bar{q}}_1^H, \dots, \bar{\bar{q}}_n^H)$ be the unique Nash equilibrium of the induced constrained game $(N^H, \bar{A}^H, \pi^H)$ and $\bar{v}(S; H) = \sum_{i \in S} \pi_i (\bar{\bar{q}}_1^H, \dots, \bar{\bar{q}}_n^H)$. We claim that $\sum_{i=1}^r \bar{v}(S_i; P) \leq \bar{v}(S; H)$, that is, $\bar{v}$ is partially superadditive. We first prove that $\bar{\bar{q}}^H \equiv \sum_{i \in N} \bar{\bar{q}}_i^H \leq \sum_{i \in N} \bar{\bar{q}}_i^P = \bar{\bar{q}}^P$. Suppose contrary to the assertion that $\bar{\bar{q}}^H > \bar{\bar{q}}^P$. Then, since $\bar{\bar{q}}_j^H \leq \bar{\bar{q}}_j^P = k_i$ for each $j \in N \backslash S$, $\sum_{i \in S} \bar{\bar{q}}_i^H > \sum_{i \in S} \bar{\bar{q}}_i^P$ and, therefore, $\sum_{j \in S_i} \bar{\bar{q}}_j^H > \sum_{j \in S_i} \bar{\bar{q}}_j^P$ for at least one non-singleton coalition S_i and for each $j \in S_i$, $c_j'(\bar{\bar{q}}_j^P) = (\sum_{k \in S_i} \bar{\bar{q}}_k^P) p'(\bar{\bar{q}}^P) + p(\bar{\bar{q}}^P) > (\sum_{k \in S_i} \bar{\bar{q}}_k^H) p'(\bar{\bar{q}}^P) + p(\bar{\bar{q}}^P) \geq (\sum_{k \in S_i} \bar{\bar{q}}_k^H) p'(\bar{\bar{q}}^H) + p(\bar{\bar{q}}^H) = c_j'(\bar{\bar{q}}_j^H)$. Thus, $\bar{\bar{q}}_j^P > \bar{\bar{q}}_j^H$, since c_i is strictly convex. But this contradicts that $\sum_{j \in S_i} \bar{\bar{q}}_j^H > \sum_{j \in S_i} \bar{\bar{q}}_j^P$. Hence, $\sum_{i \in S} \bar{\bar{q}}_i^H \leq \sum_{i \in S} \bar{\bar{q}}_i^P$ and $\bar{\bar{q}}^H \leq \bar{\bar{q}}^P$. This also implies that $\bar{\bar{q}}_j^H = \bar{q}_j = k_j$ for all $j \in N \backslash S$. To see this, suppose on the contrary that $\bar{\bar{q}}_j^H < \bar{q}_j$ for some $j \in N \backslash S$. Then, since $\bar{\bar{q}}^H \leq \bar{\bar{q}}^P \leq \bar{q}$ (as shown), $c_j'(\bar{\bar{q}}_j^H) =$

[11] This too is analogous to a result in a climate change model (Chander and Tulkens, 1997: Proposition 4).

$p(\bar{\bar{q}}^H) + \bar{\bar{q}}_j^H p'(\bar{\bar{q}}^H) \geq p(\bar{q}) + \bar{\bar{q}}_j^H p'(\bar{q}) > p(\bar{q}) + \bar{q}_j p'(\bar{q}) = c_j'(\bar{q}_j) \Longrightarrow \bar{q}_j < \bar{\bar{q}}_j^H$, which contradicts our supposition that $\bar{\bar{q}}_j^H < \bar{q}_j$. Therefore, $\bar{\bar{q}}_j^H = \bar{q}_j = k_i$ for all $j \in N\backslash S$. Since coalition S could have chosen for each $i \in S$ an output level $\bar{\bar{q}}_i^P$ but chose instead $\bar{\bar{q}}_i^H$ and $\bar{\bar{q}}_j^H = \bar{\bar{q}}_j^P = \bar{q}_j = k_j$ for each $j \in N\backslash S$, it follows that $\bar{v}(S; H) = \sum_{i \in S} \pi_i (\bar{\bar{q}}_1^H, \dots, \bar{\bar{q}}_n^H) \geq \sum_{i=1}^r \sum_{j \in S_i} \pi_j (\bar{\bar{q}}_1^P, \dots, \bar{\bar{q}}_n^P) = \sum_{i=1}^r \bar{v}(S_i, P)$, i.e. the partition function game $(N, \bar{v})$ is partially superadditive. ■

Proposition 8 provides useful insights into the industry structure that is likely to emerge in the absence of an antitrust policy when the firms face capacity constraints equal to their Nash equilibrium outputs, as it implies that *all* existing cartels (i.e. non-singleton coalitions) in the industry will have incentives to merge and form a *single* cartel followed by a competitive fringe. As seen from the proof of Proposition 8, the total output will fall after the merger but the total profit/payoff of the firms in the merger of all cartels will be higher and so will be the profits of the outside firms. Comparing propositions 7 and 8, it follows that while merger of all existing cartels in an industry is profitable, mergers among two or more standalone firms to form a single cartel may or may not be profitable as compared to their Nash equilibrium payoffs.

It is noteworthy that propositions 7 and 8, unlike Proposition 6, do not require the firms to be identical, but require the capacity constraints to be equal to their Nash equilibrium outputs. Restriction to these capacity constraints plays a crucial role in making the corresponding partition function game partially superadditive. For higher capacity constraints the standalone firms may produce so much more that the merger of all existing cartels will no longer be profitable, as seen in Example 1. Thus, merger of all cartels may become unprofitable overtime as the outside firms expand their capacities to produce more.

It may be also noted that if the firms are sufficiently heterogeneous, then the strong-core payoff vectors may require, as do the γ-core payoff vectors in a different context (Chander and Tulkens, 1997), transfers between firms to balance the gains and losses from the merger to monopoly: for some firms the gains from higher prices may be higher than their losses due to their lower production and for others the opposite may be the case.[12]

[12] However, mergers that require transfers between firms may be easier to detect and prevent.

Proposition 8 can be shown to hold also for capacity constraints that are lower than the Nash equilibrium outputs, but not "too low". This is because capacity constraints that are lower than the Nash equilibrium outputs but are not too low would also be binding for the outside firms but not for the firms in the merged cartels. However, if they are too low, then they become binding also for some or all firms in the merged cartels and Proposition 8 can no longer be shown to hold.

4. An additional interpretation of the strong-core

Since the strong-core seems to be of general interest, I offer an additional interpretation of the strong core. This additional interpretation comes from the fact that partitions in which some non-singleton coalition is worse-off relative to a strong-core payoff vector are not stable in the sense that they are "farsightedly" dominated by the strong-core payoff vector itself. This means that in any partition that may possibly form subsequent to a deviation either the deviating coalition is immediately worse-off or farsightedly not better-off. We now make this precise by formally introducing a notion of farsighted dominance in partition function games.

4.1 *Farsighted dominance*

A central idea underlying the notion of farsighted dominance is that the players may form a new partition from an existing one and each coalition in the new partition may freely and independently choose its part of the feasible payoff vector. More specifically, coalitions in a partition may split or merge to form a new partition. Some of the coalitions involved in this process may be thought of as "perpetrators" in the formation of the new partition and others as "residual" coalitions of players left behind by the perpetrators. Formally, let $P = \{S_1, \dots, S_m\}$ be an existing partition, then P' is a partition formed from P if there is a coalition T such that $P' = \{T, S_1\backslash T, \dots, S_m\backslash T\}$ where coalition T is the perpetrator and coalitions $S_i\backslash T, i = 1, \dots, m$, are the residuals. We shall denote formation of a partition P' with a feasible payoff vector y' from an existing partition P with a feasible payoff vector y by

$$(y, P) \underset{T}{\rightarrow} (y', P'),$$

where T is the perpetrator and y' is the feasible payoff vector chosen independently by the coalitions in the new partition P' – the perpetrator T, like other coalitions in the partition P', chooses only its part of y'. Since the worth of a coalition in a partition function game depends on the entire partition, the worth of even those coalitions that are "untouched" by the move of perpetrator T may change, necessitating adjustments, by them, even in their parts of the feasible payoff vector. It is worth considering the two extreme cases: a player leaves a coalition and forms a singleton coalition, i.e., $|T| = 1$ or all coalitions merge, i.e., $T = \cup_{i=1}^{m} S_i = N$ and, thus, each $S_i \backslash T = \emptyset, i = 1, \dots, m$. In the latter case, I shall follow the convention that $\{N, \emptyset, \dots, \emptyset\} \equiv \{N\}$.

Definition 5 A payoff vector (y, P), $P \neq N$, is farsightedly dominated by a payoff vector (x, N) if there is a sequence of payoff vectors $(y^0, P^0), (y^1, P^1), \dots, (y^q, P^q)$, where $(y^0, P^0) = (y, P)$ and $(y^q, P^q) = (x, N)$, and a sequence of coalitions T^h such that for each $h = 1, \dots, q$: $(y^{h-1}, P^{h-1}) \underset{T^h}{\rightarrow} (y^h, P^h)$ and $x_i \geq y_i^{h-1}$ for each $i \in T^h$.

In words, there could be several steps in moving from the feasible payoff vector $(y, P), P \neq N$, to the feasible payoff vector (x, N). Since, by our convention, each player prefers to be a member of the grand coalition than of a coalition in a partition other than the grand coalition even if its payoff is the same, farsighted dominance requires that every member of each coalition that makes a move at some step must be better-off at the end of the process. What matters to the members of coalitions involved in moving the process are their "final" payoffs – and not their payoffs at intermediate stages.[13]

The above notion of farsighted dominance is similar to that in Harsanyi (1974), but differs in two important respects. First, it is defined for a partition function game whereas Harsanyi's notion of farsighted dominance and its elegant modification, as motivated and proposed by Ray and Vohra (2015), are both defined for characteristic function games, which as noted above are a special case of partition function games. Second, like the modification by Ray and Vohra (2015), it is both non-coercive and respects coalitional sovereignty, since by definition of a feasible payoff vector for a partition, $\sum_{i \in S} y_i^h = v(S; P^h)$ for each $S \in P^h, h = 1, \dots, q$, and each coalition in each partition P^h in the sequence is free to decide independently its part of the payoff vector y^h. [14] A third point is that the definition can be extended to allow

[13] See Page Jr. et al. (2005) for a related but different concept of farsighted dominance in which payoffs at intermediate stages matter.

[14] In contrast, Harsanyi's (1974) notion of farsighted dominance, as Ray and Vohra (2015) note, is both coercive and violates coalitional sovereignty.

for more than one perpetrator at each step of the sequence. All results below hold in this case as well, but for concreteness I assume that there is only one perpetrator at each step.

Proposition 9 Let (x, N) be a strong-core payoff vector for a partition function game (N, v). Then every feasible payoff vector $(y, P), P \neq N$, that may possibly be chosen subsequent to a deviation from (x, N), is farsightedly dominated by (x, N).

Proof: I first prove that if $P = [N]$, then $x_i \geq v(i; [N]), i = 1, \dots, n$. Suppose the finest partition $[N]$ is formed subsequent to a deviation by a singleton coalition $\{i\}$. Then since (x, N) is a strong-core payoff vector and no coalition in the finest partition is a non-singleton, it follows that the deviating coalition $\{i\}$ itself must be worse-off, i.e., $v(i; [N]) \leq x_i$. Since this is true for every i, it follows that $x_i \geq v(i; [N]), i = 1, \dots, n$.

Let $P = \{S_1, \dots, S_m\} \neq N$ be a partition formed subsequent to a deviation by a (singleton or non-singleton) coalition and let $(y^0, P^0) \equiv (y, P)$. If $P = [N]$, then $(y^0, P^0) \underset{T^1}{\rightarrow} (y^1, P^1) \equiv (x, N)$, where $T^1 = \cup_{j=1}^{m} S_j = N$, and $y_i^1 = x_i \geq y_i^0 = y_i$ for each $i \in T^1 = N$, since $x_i \geq v(i, [N]) = y_i, i = 1, \dots, n$. Thus, (x, N) farsightedly dominates (y, P), if $P = [N]$. If $P \neq [N]$, then P^0 includes at least one non-singleton coalition which is worse-off and, thus, at least one member of the coalition is worse-off, i.e., $y_i^0 \leq x_i$ for at least some i. Let $T^1 = \{i\}$, $P^1 = \{T^1, S_1 \backslash T^1, \dots, S_m \backslash T^1\}$, and y^1 be a feasible payoff vector for the partition P^1. If $P^1 = [N]$, then $(y^0, P^0) \underset{T^1}{\rightarrow} (y^1, P^1) \underset{T^2}{\rightarrow} (y^2, P^2) = (x, N)$, where $T^2 = T^1 \cup \cup_{j=1}^{m} S_j \backslash T^1 = N$, and $y_i^0 \leq x_i$ and $y_j^1 \leq x_j, j \in T^2 = N$, since $x_i \geq v(i, [N]) = y_i^1, i = 1, \dots, n$. Thus, (x, N) farsightedly dominates (y, P). If $P^1 \neq N$, then, proceeding similarly, there exists a sequence $(y^0, P^0) \underset{T^1}{\rightarrow} (y^1, P^1) \underset{T^2}{\rightarrow} (y^2, P^2) \underset{T^3}{\rightarrow} \dots \underset{T^{q-1}}{\longrightarrow} (y^{q-1}, P^{q-1}) \underset{T^q}{\rightarrow} (y^q, P^q) = (x, N)$, where $|T^h| = 1, h = 1, \dots, q-1, T^q = N$, and $y_j^h \leq x_j, j \in T^h, h = 1, \dots, q$. Thus, (x, N) farsightedly dominates (y, P). ■

The proposition implies that no partial cartel in an industry may be stable if the firms are farsighted and conservative. An example can help illustrate the role played by farsightedness in the determination of the strong-core.

Example 2 Let $N = \{1,2,3,4\}$ and $v(S; P) = |S|^3(|N| - |P|), S \subset N$.

In this game, $v(N; N) = 192$, $v(i; [N]) = 0$, and it is easily seen that the feasible payoff vector $(0,64,64,64)$ belongs to the strong-core. In particular, if coalition $\{1\}$ deviates from $(0,64,64,64)$, then at least one non-singleton coalition is worse-off in each partition $P \neq N$ of which $\{1\}$ is a member: $v(N\backslash 1; \{1, N\backslash 1\}) = 54 < 64 + 64 + 64$ (since $|N\backslash 1| = 3$ and $|P| = 2$) and for every two-player coalition $S \subset \{2,3,4\}$, we have $v(S; \{1, S, N\backslash S\backslash 1\}) = 8 < 64 + 64$ (since $|S| = 2$ and $|P| = 3$). Finally, coalition $\{1\}$ itself is worse-off in the finest partition, since its payoff in the finest partition is the same as in the payoff vector $(0,64,64,64)$ that is feasible for the grand coalition and, by our convention, a coalition is worse off in a partition other than the grand coalition if its payoff is the same as in the grand coalition. Similarly, for deviations by other coalitions from $(0,64,64,64)$.

It is worth noting that if the singleton coalition $\{1\}$ deviates from the feasible payoff vector $(0,64,64,64)$, it cannot be sure that partition $\{1, N\backslash 1\}$ will surely form and the payoff will be according to the payoff vector $(2, 18,18,18)$ that is feasible for $\{1, N\backslash 1\}$. This is because the partition $\{1, N\backslash 1\}$ is not stable and the payoff vector $(2, 18,18,18)$ is farshightedly dominated by the strong-core payoff vector $(0,64,64,64)$. In other words, if coalition $\{1\}$ deviates from the grand coalition and the payoff vector is $(0,64,64,64)$, then since the residual coalition $\{2,3,4\}$ is worse-off, it can deter the deviation by $\{1\}$ by first breaking apart, resulting in payoffs $(0,0,0,0)$, and then proposing the payoff vector $(0,64,64,64)$ that is feasible for the grand coalition and a strict Pareto improvement over $(0,0,0,0)$, including for $\{1\}$, since by our convention a coalition is better-off in the grand coalition even if its payoff is the same as in a coalition in a partition other than the grand coalition.

5. Conclusion

From a game theory perspective, we have introduced a new core concept for partition function games and established its many properties. Unlike the previous core concepts, the new concept, labelled the strong-core, makes no *ad hoc* assumption regarding the coalitions that may form subsequent to a deviation from the grand coalition. Thus, it seems to settle a long standing debate on which core concept to use for partition function games.

It was shown that the strong-core is nicely related to the familiar γ- and δ- cores in that for games with positive externalities, δ-core $\subset$ strong-core $\subset$ γ-core; for games with negative externalities, strong-core = γ-core $\subset$ δ-core; and for games in general, strong-core $\subset$ γ-core.

Apart from showing that the strong-core is consistent with the familiar γ- and δ- cores, these inclusion relationships are important also because they have implications for the existence of a nonempty strong-core. In particular, they imply that for games with positive (negative) externalities, the strong-core is nonempty if the δ-core (γ-core) is nonempty; and non-emptiness of the γ-core is both a necessary and sufficient condition in games with negative externalities and a necessary condition for non-emptiness of the strong-core in games in general. Since existence of a non-empty δ- or γ- core can be established by applying the Bondareva-Shapley theorem (Bondareva,1963 and Shapley, 1967) and in applications most games exhibit either positive or negative externalities, the inclusion relationships and the Bondareva-Shapley theorem can be used to prove existence of a non-empty strong-core. Although establishing existence of a nonempty strong-core is undeniably important, it may be noted that an empty strong-core is also of significance for it implies that the grand coalition is unlikely to be formed and sustained.

I have focused on an application to a Cournot oligopoly because almost every study on partition function games or games with externalities invariable refers to this game. But as mentioned in fn.7, the strong-core has additional applications. I do not pursue the additional applications here as they require different models and leave them for a future research project.

From an industrial organization perspective, the analysis of an oligopoly in this paper complements Salant et al. (1983) in that it is concerned with stability of cartels, especially of the merger-to-monopoly, which has received little attention in the literature so far. The basic approach in the paper is the same as in Salant et al. – only the question addressed is different. We have shown that firms in a Cournot oligopoly not only have incentives to merge to monopoly, but the merger is also stable in both long and short runs, since a non-empty strong-core is a sufficient condition for stability of the grand coalition. This was shown in a model with general demand and cost functions and any finite number of firms. I assumed strictly convex cost functions because some scholars justifiably consider constant marginal costs a relatively less interesting case (see e.g. Perry and Porter, 1985). However, the analysis in this paper can be shown to hold also in the case of identical constant marginal costs. This is because with constant marginal costs forming cartels/coalitions is less profitable for the cartels but more profitable for the standalone firms outside the cartels, implying lower profits/payoffs for cartels/coalitions and thus raising the possibility of a nonempty strong-core.

It was shown that the merger-to-monopoly is stable also if the oligopolistic firms face specific capacity constraints that are not only natural and less arbitrary, but also lead to the interesting result that the merger of all existing cartels when the firms, inside or outside the cartels, cannot expand their production capacities will lower the total industry output but lead to higher profits for the firms both inside and outside the cartels because of higher prices. However, the merger of all cartels may become unprofitable over time, as the firms outside the cartels may expand their production capacities.

Appendix

Proof of Lemma 5: Since each $\pi_i(.)$ is concave and continuous in $q_1, \dots, q_n$ and each A_i is compact and convex, the game (N, A, π) admits a Nash equilibrium $(\bar{q}_1, \dots, \bar{q}_n)$. Suppose contrary to the assertion that the game has another Nash equilibrium, say $(\bar{\bar{q}}_1, \dots, \bar{\bar{q}}_n)$, and $(\bar{q}_1, \dots, \bar{q}_n) \neq (\bar{\bar{q}}_1, \dots, \bar{\bar{q}}_n)$. Without loss of generality, let $\bar{q} = \sum_{i \in N} \bar{q}_i \geq \sum_{i \in N} \bar{\bar{q}}_i = \bar{\bar{q}}$. Since $(\bar{q}_1, \dots, \bar{q}_n) \neq (\bar{\bar{q}}_1, \dots, \bar{\bar{q}}_n)$, $\bar{q}_i > \bar{\bar{q}}_i$ for at least one i. Furthermore, $p'(\bar{\bar{q}})\bar{\bar{q}}_i + p(\bar{\bar{q}}) > p'(\bar{\bar{q}})\bar{q}_i + p(\bar{\bar{q}}) \geq p'(\bar{q})\bar{q}_i + p(\bar{q})$, since $\bar{q} \geq \bar{\bar{q}}$ and by assumption the marginal revenue of each firm is non-increasing with total demand q. From the first order conditions for a Nash equilibrium $c_i'(\bar{\bar{q}}_i) = p'(\bar{\bar{q}})\bar{\bar{q}}_i + p(\bar{\bar{q}}) > p'(\bar{q})\bar{q}_i + p(\bar{q}) = c_i'(\bar{q}_i)$ implying $\bar{q}_i < \bar{\bar{q}}_i$, which is a contradiction. ■

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